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Simulating the fourth-order interference phenomenon of anyons with photon pairs

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Abstract. The generalized Hong-Ou-Mandel interferometer with anyons is studied. Novel interference results different from bosons or fermions are found. An experimental scheme based on linear optics is proposed and realized to simulate the fourth-order interference phenomenon of anyons.

PACS. 42.50.Xa Optical tests of quantum theory – 03.65.Vf Phases: geometric; dynamic or topological – 05.30.Pr Fractional statistics systems (anyons, etc.)

QICS. 03.60.+i Entanglement of identical particles and statistics

In 1970s, scientists [1] found that in the two-dimensional world there exist quasi-particles that exhibit fractional quantum statistics. Different from bosons and fermions, a phase shift $e^{i\pi\theta}$ is created if two of these type of particles are interchanged; also a phase shift of $e^{i2\pi\theta}$ is created if such type of particle runs over a closed loop. Here, θ may vary from zero to 1. Such type of particle was first dubbed by Wilczek [2] with the name anyon, which was then paid lots of attention by many scientists [3–10], not only because of its exotic behavior, but also because of its topological property that makes it robust against decoherence error. This robustness makes it a desirable candidate for qubits in fault-tolerant quantum computing [8,11–13].

As it is well-known, one has to perform quantum measurements in order to do quantum computation. So far, some methods based on anyons have been proposed to perform quantum measurements [14,15]. Most generally, one can look at the interference results with these quasi-particles [11,16–20]. However, the existing proposals have mainly focused on the interference of a single anyon. Here we propose an idealized experiment on the interference of entangled anyons, which is a generalized Hong-Ou-Mandel (HOM) interferometer [21] and it can be useful in constructing a quantum computer in the future.

In what follows we shall first present the theoretical picture of interference phenomenon of bi-anyons compared with bi-bosons and bi-fermions. We then report an optical experiment to simulate the interference phenomenon of bi-anyons with photon pairs. Finally, we conclude our results.

The annihilation and creation operators a_i and $a_i^\dagger$ for bosons in mode i obey the commutation relation [22]

$$a_i^\dagger a_j^\dagger = a_j^\dagger a_i^\dagger, a_i a_j = a_j a_i, \text{ and } a_i a_j^\dagger - a_j^\dagger a_i = \delta_{ij} \ \forall \ i,\ j. \tag{1}$$

And the annihilation and creation operators a_i and $a_i^\dagger$ for fermions in mode i obey the anti-commutation relation

$$a_i^\dagger a_j^\dagger = -a_j^\dagger a_i^\dagger, a_i a_j = -a_j a_i, \text{ and } a_i a_j^\dagger + a_j^\dagger a_i = \delta_{ij} \ \forall \ i,\ j. \tag{2}$$

While for anyons, since interchange of two of these particles can give any phase, the annihilation and creation operators a_i and $a_i^\dagger$ for anyons in mode i obey the relation

$$a_i^\dagger a_j^\dagger = e^{i\epsilon} a_j^\dagger a_i^\dagger, a_i a_j = e^{-i\epsilon} a_j a_i, \\ \text{and } a_i a_j^\dagger - e^{i\epsilon} a_j^\dagger a_i = \delta_{ij} \ \forall i, j. \tag{3}$$

Here i and j refer to the spatial degrees of freedom of the particles.

For convenience, we first consider the simplest kind of HOM interference with two particles.

In Figure 1, the transformation matrix of the beam-splitter (BS) is $U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$ and the input state is

$$|\Phi_{in}\rangle = a_1^\dagger a_2^\dagger |0\rangle ,$$

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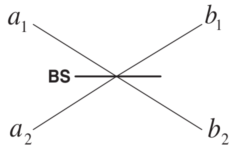

Fig. 1. Particles with modes a_1 and a_2 meet at a 50/50 beam-splitter and the output mode are b_1 and b_2.

After the beam-splitter, the system is prepared into the state

$$|\Phi_{out}\rangle = \frac{1}{2}\left(ib_1^\dagger + b_2^\dagger\right)\left(b_1^\dagger + ib_2^\dagger\right)|0\rangle$$
$$= \frac{1}{2}\left\{i\left[\left(b_1^\dagger\right)^2 + \left(b_2^\dagger\right)^2\right] + \left(b_2^\dagger b_1^\dagger - b_1^\dagger b_2^\dagger\right)\right\}|0\rangle . \tag{4}$$

Bosons obey the commutation law (1), therefore the output state becomes

$$|\Phi_{out}\rangle = \frac{1}{2}\left[\left(b_1^\dagger\right)^2 + \left(b_2^\dagger\right)^2\right]|0\rangle . \tag{5}$$

This implies a zero-coincidence count at the output ports, it is the famous HOM dip. Fermions obey the anti-commutation law (2), so the output state is

$$|\Phi_{out}\rangle = b_1^\dagger b_2^\dagger |0\rangle . \tag{6}$$

It is the HOM peak. While for anyons, they obey the relation (3), therefore the output state is

$$|\Phi_{out}\rangle = \frac{1-\mathrm{e}^{-i\epsilon}}{2} b_1^\dagger b_2^\dagger |0\rangle . \tag{7}$$

The corresponding probability for observing coincidence counts is

$$P = \frac{1-\cos\epsilon}{2}. \tag{8}$$

It can vary between 0 and 1 with different types of anyons. When $\epsilon = 0$, it becomes a Boson type interference, when $\epsilon = \pi$, it is a Fermion type interference. Therefore, the HOM interference can be generalized to anyons.

Figure 2 shows the theoretical interference curves of bosons, fermions and anyons.

The phenomena of bunching (bosons) and anti-bunching (fermions) can be observed experimentally with existing lab conditions, because there exist real bosons and fermions in the nature. However, the idealized interference of anyons is hard to realize experimentally, because anyons can only exist in two dimensional world, therefore it is very demanding for real experimental observation. In order to carry out some researches on the characters of anyons, we design an optical experimental scheme, which can result in similar interference phenomenon as anyons.

Figure 3 is our experiment setup. A continuous wave (CW) laser is used to pump a piece of type I BBO ($\beta - \mathrm{BaB_2O_4}$) crystal (1 mm). The down conversion photon pairs ($|\mathrm{HH}\rangle$) propagate along path 1 and path 2 respectively. After being reflected by mirrors, they meet at a

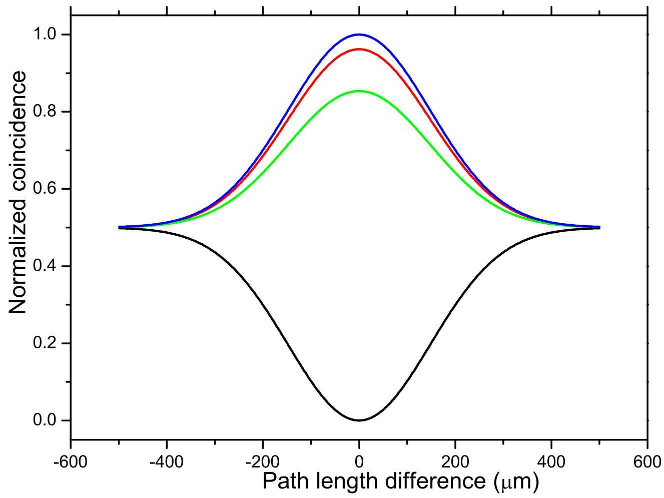

Fig. 2. (Color online) The generalized HOM interference curves of bosons, fermions, and anyons. The curve on the top corresponds to $\epsilon = \pi$ (fermions), the one on the bottom corresponds to $\epsilon = 0$ (bosons), and those curves between these two from up to down correspond to $\epsilon = 7\pi/8, \pi/2$ (anyons) respectively.

50/50 beam-splitter (BS), and a half wave plate (HWP1) is inserted into path 2 before the BS. Then photons coming from port 3 and port 4 orderly go through a series of wave plates severally. It consists of phase plate, half wave plate (HWP2), polarization beam-splitter (PBS) (which allows only horizontal polarization to pass.) and interference filter. Finally, they are registered by single photon detectors D1 and D2. The position of mirror 2 can be changed by a step motor to observe the HOM interference.

Operation details are listed in the following.

Firstly, HWP1 and HWP2 are all set at zero degree, and the phase plates are removed away. The corresponding interference curve is a normal HOM dip.

Secondly, HWP1 and HWP2 are both set at 22.5° simultaneously. Then a pair of phase plates are inserted according to positions in Figure 3, with their fast axes set orthogonally. It can generate a delay phase ϕ_1 between horizontal (H) and vertical (V) polarization in path 4 and a forward phase ϕ_1 between H and V components in path 3.

After the final projecting measurement, the corresponding HOM interference curve can be attained.

For clarity, we now give a mathematical picture for the whole process. Initially, the state of down conversion photon pairs is $|\mathrm{HH}\rangle_{12}$. After HWP1, it becomes $|\mathrm{HV}\rangle_{12}$. Then after the transformation of beam splitter (the BS which we used is plated and can produce a π phase difference between H and V components when reflecting), it is in the state $\frac{1}{2}i(|\mathrm{HV}\rangle_{33} + |\mathrm{HV}\rangle_{44}) - \frac{1}{2}(|\mathrm{HV}\rangle_{34} - |\mathrm{HV}\rangle_{43})$. Because the terms $|\mathrm{HV}\rangle_{33}$ and $|\mathrm{HV}\rangle_{44}$ will never cause the twofold coincidence of detectors D1 and D2, we can neglect them and only consider the terms $|\mathrm{HV}\rangle_{34}$ and $|\mathrm{HV}\rangle_{43}$. After normalization, it becomes the state $\frac{1}{\sqrt{2}}(|\mathrm{HV}\rangle_{34} - |\mathrm{HV}\rangle_{43})$, which consists of reflecting and transmitting components.

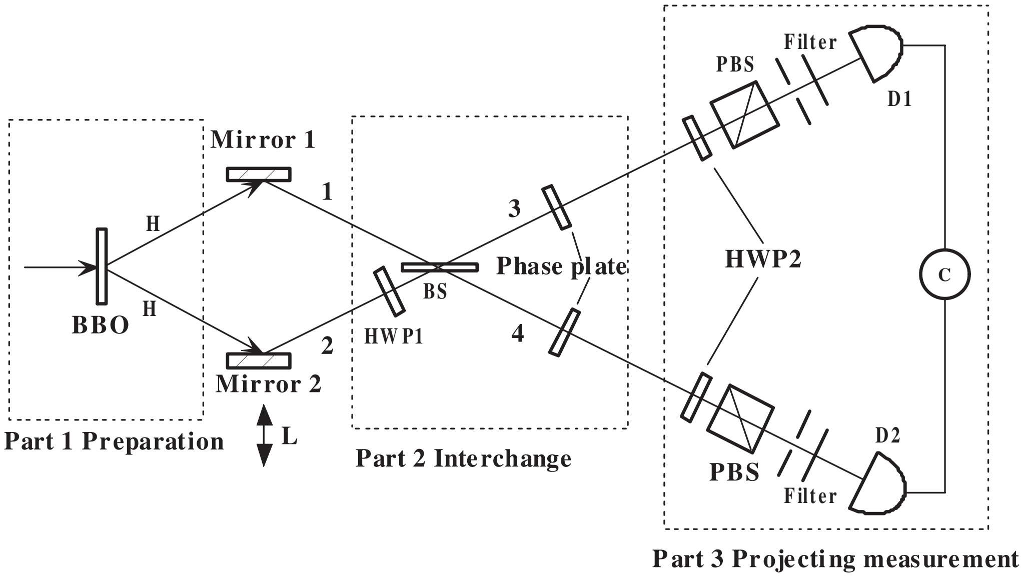

Fig. 3. Experimental setup of the simulating scheme. It is divided into three parts. The first part is state preparation, in which a CW laser is used to pump a piece of BBO ($\beta-\mathrm{BaB_2O_4}$) crystal to produce the spontaneous parametric down conversion. The second part is interchange, which is made up of HWP1, BS and a pair of phase plates. The third part is projecting measurement, which consists of HWP2, PBS, interference filters and detectors.

In the following, when passing through the phase plates (which can generate a difference phase ϕ_1 between reflecting and transmitting components), it evolves into the state $\frac{1}{\sqrt{2}}(|\mathrm{HV}\rangle_{34} - \mathrm{e}^{2i\phi_1}|\mathrm{HV}\rangle_{43})$. Furthermore, a projecting measurement (on $(|\mathrm{H}\rangle+|\mathrm{V}\rangle)_3(|\mathrm{H}\rangle+|\mathrm{V}\rangle)_4$ basis) is carried out. Now the reflecting and transmitting components are indistinguishable, so they can result in interference. And the probability of the measurement is

$$P = \frac{1-\cos(2\phi_1+\pi)}{2}. \tag{9}$$

The experimental interference curves are shown in Figure 4. Curves (1, 2, 3, 4) correspond to insert a pair of phase plates $(0, \pi/16, \pi/8, \pi/2)$ respectively. We find that, they show similar interference phenomenon like those in Figure 2. Therefore, the HOM interference curve of any bi-anyons can be simulated with bi-photons by inserting a pair of proper phase plates in Figure 3.

In conclusion, we have studied the generalized HOM interference of bi-anyons. They may perform exotic behavior compared with bosons and fermions. Though there does not exist real anyons in the nature, we can still study some of their properties through simulating experiments, such as the property of a fractional phase after interchanging two of these particles. In our simulating experiment, what we have done is importing an additional phase between two particles by inserting phase plates after the beam splitter, and indicating it with a HOM interference curve. Of course, the manipulation on the photons is basically a two-photon interference effect. Furthermore, it can be extended to the interference of multi-anyons, which will also result in interesting phenomenon. Hopefully, our simulating experiment will provide a new avenue to carry out some researches on anyons.

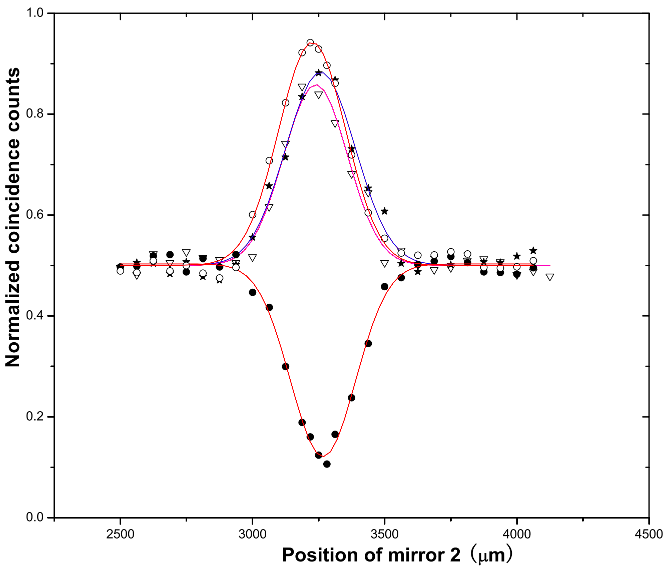

Fig. 4. (Color online) Experimental HOM interference curves. The x-axis is the path length difference between path 1 and path 2, and the y-axis is the probability of coincidence counts. Curves from up to down (1, 2, 3, 4) correspond to insert a pair of phase plates $(0, \pi/16, \pi/8, \pi/2)$ respectively.

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